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# Effects of variable viscosity on Three-Dimensional Boundary Layer Flow of Non-Newtonian Fluids over a Stretching Surface with Mass and Heat Transfer

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**Abstract**— Three dimensional flow of non-Newtonian viscoelastic fluid with the variation in the viscosity over a stretching surface is investigated. The governing partial differential equations of continuity, momemtum, energy and concentration are transformed into non-linear ordinary differential equations by using similarity transformations. The transformed equations are solved numerically by fourth-order Runge-Kutta shooting method. The effects of viscosity, heat and the stretching ratio parameters with Prandtle and Schmidt numbers on the velocity, temperature and concentration distributions have been discussed and illustrated graphically.

**Index Terms**— Boundary layer, heat transfer, non-newtonian , stretching surface, three-dimensional flow, thermal conductivity , Variable viscosity, Visco-elastic fluid.

## 1 Introduction

THE study of flow over a stretching surface has generated much interested in recent years in view of its numerous industrial applications such as extrusion of polymer sheets, continuous casting, glass blowing, rolling and manufacturing plastic flims and artificial fibers. Sakiadis [1] was probably the first to study the two-dimensional boundary layer flow due to a stretching surface in a fluid at rest. An extension of the problem to the case of suction or injection at the surface was investigated by Ericson et al [2]. Crane [3] and Ali [4] carried out a study for a staeching surface subject to suction or injection for uniform and variable surface temperatures. Chakrabarti and Gupta [5] studied the temperature distribution in this MHD boundary layer flow over a stretching sheet in the presence of suction. There are several extensions to this problem, which include consideration of more general stretching velocity and the heat transfer.

The heat transfer in flow over a stretching surface was investigated by Gupta and Gupta et al. [6], where the surfaces held at constant temperature and is subject to suction and blowing.

Interest of researchers in the flows of non-Newtonian fluids in the presence of heat transfer have relevance in food enggineering, petroleum production, power engineering and in industrial processes including polymer melt and polymer solutions used in the plastic processing industries. Some recent studies of the topic can be seen in Labropulu et al [7], Sahoo [8], Mustafa et al [9], Hayet et al [10]. It is noted that most studies in the literature discussed the two-dimensional boundary layer flows.

The problem of three-dimensional boundary layer flow of a viscoelastic fluid due to a stretching surface, has been considered before as Hayet et al [11] without mass and heat transfer. Fox et al [12] used both exat and approximate methods to examine the boundary layer flow of a viscoelastic fluid characterized by a power law model. Vajravelu and Rollins [13] investigated the heat transfer of the boundary layer flow of second grade fluid. Mahantesh et al [14] discussed the flow and heat transfer charecteristics of a viscoelastic fluid in a porous medium over an impermeable stretching sheet with viscous dissipation.

In most of the studies of this type of problems, the viscosity and thermal conductivity of the ambient fluid were assumed to be constant. However, Hussanien et al [15] revealed that the fluid viscosity and thermal conductivity might function of temperatures. It is known that these physical properties can change significantly with temperature and when the effects of variable viscosity and thermal conductivity are taken in to account, the flow charecteristics are significantly changed compered to the constant property case.

In this paper, we investigate the steady three dimensional laminar boundary layer flow of viscous incompressible second grade fluid over a stretching surface, when the viscosity and the thermal conductivity are function of temperature. By em-

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ploying the similarity transformation, the boundary layer equations governing the flow are reduced to ordinary differential equations and solved numerically using fourth-order Runga-Kutta shooting method.

## 2 Mathematical Formulation

We consider the steady three dimensional flow of a viscous incompressible second grade fluid bounded by a stretching surface. Under the usual boundary layer approximations the flow is governed by the following equations:

The equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

The equation of momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)+k_0\left[u\frac{\partial^3 u}{\partial x\partial z^2}+w\frac{\partial^3 u}{\partial z^3}-\left(\begin{array}{l}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2}+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}\\ +2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+2\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\end{array}\right)\right] \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right)+k_0\left[v\frac{\partial^3 v}{\partial y\partial z^2}+w\frac{\partial^3 v}{\partial z^3}-\left(\begin{array}{l}\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial z^2}+\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z^2}\\ +2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial y\partial z}+2\frac{\partial w}{\partial z}\frac{\partial^2 v}{\partial z^2}\end{array}\right)\right] \tag{3}$$

The energy equation:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=k\frac{\partial^2 T}{\partial z^2}+Q\left(T-T_\infty\right) \tag{4}$$

And the concentration equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D\frac{\partial^2 C}{\partial z^2}+\kappa_1\left(C-C_\infty\right) \tag{5}$$

Where the velocity components in the $x-$, $y-$ and $z-$ directions are denoted by $u$, $v$ and $w$ respectivly, $\rho$ is the fluid density, $\upsilon=\frac{\mu}{\rho}$ is the Kinemetic viscosity, $\mu$ is the viscosity, $k_0$ is the material parameter, $C_p$ is the specific heat at constant pressure, $k$ is the thermal conductivity, $T$ is temperature of the fluid flow, $C$ is the mass concentration of the species of the flow, $Q$ is the volumetric rate of heat generation/absorption, $D$ is the molecular diffusion coefficient, $T_\infty$ and $C_\infty$ are the fluid temperature and concentration far away and $\kappa_1$ is the reaction rate coefficient.

In most of the studies, of this type of problems, the viscosity and thermal conductivity of the fluid were assumed to be constant. However, it is known that physical properties can change significantly with temperature and when the effects of variable viscosity and thermal conductivity are taken in to account, the flow characteristics are substantially changed compared to the constant property case. Hence in the problem under consideration, the viscosity and thermal conductivity have been assumed to be inverse linear functions of temperature. We assume,

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma\left(T-T_\infty\right)\right] \quad \text{or} \quad \frac{1}{\mu}=b_1\left(T-T_e\right) \tag{6}$$

Where $b_1=\frac{\gamma}{\mu_\infty}$ and $T_e=T_\infty-\frac{1}{\gamma}$

$$\frac{1}{k}=\frac{1}{k_\infty}\left[1+\kappa\left(T-T_\infty\right)\right] \quad \text{or} \quad \frac{1}{k}=c_1\left(T-T_r\right) \tag{7}$$

Where $c_1=\frac{\kappa}{k_\infty}$ and $T_r=T_\infty-\frac{1}{\kappa}$

Where $b_1$ , $c_1$ , $T_e$ and $T_r$ are constants and their values depend on the reference state and thermal properties of the fluid i.e. $\gamma$ and $\kappa$ . In general $b_1 > 0$, for liquids and $b_1 < 0$ for gases ( the viscosity and thermal conductivity of liquid/gas usually decrease/increase with increasing temperature).

The appropriate boundary conditions for the present problem are given by:

$$u=u_w\left(x\right)=ax, \quad v=v_w\left(y\right)=by, \quad w=0 \quad \text{at} \quad z=0$$
( $a$ , $b$ are positive constants) (8)

$$\theta\left(\eta\right)=\frac{T-T_\infty}{T_w-T_\infty}, \quad \phi\left(\eta\right)=\frac{C-C_\infty}{C_w-C_\infty} \quad \text{as} \quad z\to\infty$$

and

the governing boundary layer three dimensional equation with temperature dependent heat generation (absorbtion) as above in equation (4) , the thermal boundary conditions, depend on the type of heating process under considerations, are considered by

$$T = T_w = T_\infty + A\left(\frac{x}{l}\right)^2 = T_\infty + B\left(\frac{y}{l}\right)^2 ;$$

$$C = C_w = C_\infty + \bar{A}x^2 = C_\infty + \bar{B}y^2 \qquad \text{at} \quad z = 0$$

$$T \to T_\infty \qquad \text{as} \quad z \to \infty \tag{9}$$

Where $A$, $B$, $\bar{A}$ and $\bar{B}$ are two constants and $l$ is the characteristic length.

In order to reduce the partial differential equations to ordinary differential equations, we use the following transformations in this study:

$$u = axf'(\eta), \qquad v = ayg'(\eta)$$

and $$w = -\sqrt{a\upsilon}\{f(\eta) + g(\eta)\} \tag{10}$$

Where

$$\eta = -\sqrt{\frac{a}{\upsilon}}z \tag{11}$$

Here $f$ and $g$ are the dimensionless stream functions, $\eta$ is the similarity variable and prime denotes the differentiation with respect to $\eta$. Using (10) and (11) , the incompressibility condition (1) is identically satisfied and Equations (2) to (5) take the dimensionless form as:

$$f''' - (f')^2 + (f+g)f'' - \left(\frac{\theta'}{\theta - \theta_e}\right)f''$$
$$-K\left[(f+g)f^{iv} + (f'' - g'')f'' - 2(f' + g')f'''\right] = 0 \tag{12}$$

$$g''' - (g')^2 + (f+g)g'' - \left(\frac{\theta'}{\theta - \theta_e}\right)g''$$
$$-K\left[(f+g)g^{iv} + (g'' - f'')g'' - 2(f' + g')g'''\right] = 0 \tag{13}$$

$$\theta'' + \Pr(f+g)\theta' - \left[\left(2\Pr(f' + g') - \alpha\right)\right]\theta = 0 \tag{14}$$

$$\phi'' + S_c(f+g)\phi' - \left[\left(2S_c(f' + g') - \gamma\right)\right]\phi = 0 \tag{15}$$

The boundary conditions (8) and (9) becomes

at $\eta = 0$

$$f = 0, \quad g = 0, \quad f' = 1, \quad g' = c, \quad \theta = 1, \quad \phi = 1 \tag{16}$$

As $\eta \to \infty$

$$f' \to 0, \quad g' \to 0, \quad f'' \to 0, \quad g'' \to 0, \quad \theta \to 0, \quad \phi \to 0 \tag{17}$$

Where the dimensionless parameters are defined as:

$\Pr = \frac{\rho\upsilon C_p}{\kappa}$ (Prandtl number)

$S_c = \frac{\upsilon}{D}$ (Schmidt number)

$\alpha = \frac{Q}{a\rho C_p}$ (Heat source/Sink parameter)

$\gamma = \frac{\kappa_1}{a}$ (Chemistry reaction parameter)

And $K = \frac{k_0 a}{\upsilon}$ is the dimensionless viscoelastic parameter, $c = \frac{b}{a}$ is the the dimensionless stretching ratio. Also $\theta_e = \frac{T_e - T_\infty}{T_w - T_\infty}$ is the parameter characterizing the influence of viscosity.

When $c = 0$, the problem reduces to the two-dimensional case ($g = 0$), given by

$$f''' - (f')^2 + ff'' - \left(\frac{\theta'}{\theta - \theta_e}\right)f'' - K\left[ff^{iv} + (f'')^2 - 2ff'''\right] = 0 \tag{18}$$

$$\theta'' + \Pr f\theta' - (2\Pr f' - \alpha)\theta = 0 \tag{19}$$

$$\phi'' + S_c f\phi' - (2S_c f' - \gamma)\phi = 0 \tag{20}$$

When $c = 1$, the problem reduces to the axi-symmetric flow, where we have ( $f = g$ ), the equation becomes

$$f''' - (f')^2 + ff'' - \left(\frac{\theta'}{\theta - \theta_e}\right)f'' - 2K\left[ff^{iv} - 2ff'''\right] = 0 \tag{21}$$

$$\theta'' + 2\Pr f\theta' - (4\Pr f' - \alpha)\theta = 0 \tag{22}$$

$$\phi'' + 2S_c f\phi' - (4S_c f' - \gamma)\phi = 0 \tag{23}$$

The boundary conditions (16) and (17) becomes

$$f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1, \tag{24}$$

$$f'(\infty) = 0, \quad g'(\infty) = 0, \quad f''(\infty) = 0,$$
$$g''(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \tag{25}$$

The parameters of engineering interest for the present problem are the local skin friction coefficients, $C_{fx}$ and $C_{fy}$ along the $x-$direction and $y-$directions respectively, which are defined as :

$$C_{fx} = \frac{\tau_{wx}}{\rho u_w^{\,2}}, \qquad C_{fy} = \frac{\tau_{wy}}{\rho u_w^{\,2}}, \tag{26}$$

Where $\tau_{wx}$ is the wall shear stress along the $x-$direction and $\tau_{wy}$ is the wall shear stress along the $y-$direction. Using equations (10), we obtain the wall skin friction coefficient in $x-$ and $y-$directions respectively as follows:

$$C_{fx} = (R_e)^{-1/2} f''(0) \quad \text{and} \quad C_{fy} = (R_e)^{-1/2} \frac{v_w}{u_w} g''(0) \tag{27}$$

Where

$$\tau_{wx} = \mu\left(\frac{\partial u}{\partial z}\right)_{z=0}, \qquad \tau_{wy} = \mu\left(\frac{\partial v}{\partial z}\right)_{z=0} \tag{28}$$

and

$R_e = {u_w x}/{\upsilon}$ is the local Reynolds number.

To assess the heat transfer ability of the medium the local Nusselt number and the local heat transfer rate are defined as:

$$Nu = \frac{x q_w}{k(T_w - T_\infty)} = -(R_e)^{1/2}\theta'(0) \tag{29}$$

Where

$$q_w = -k\left(\frac{\partial T}{\partial z}\right)_{z=0} \tag{30}$$

## 3 Results and Discussion

The system of differential equations (12) to (15) governed by boundary conditions (16) and (17) are solved numerically by applying an efficient numerical technique based on the fourth order Runge-Kutta shooting method and an iterative method. It is experienced that the convergence of the iteration process is quite rapid.

The purpose of this study is to bring out the effects of the variable viscosity on the governing flow with the combinations of the other flow parameters. The three dimensional flow of the present problem is governed by seven parameters, namely $K$, the viscoelastic parameter, $\alpha$ the heat parameter, $\gamma$ the chemical reaction parameter, $c$ the dimensionless stretching ratio, $\Pr$ the Prandtl number, $S_c$ the Schmidt number and $\theta_e$ the dimensionless viscosity parameter. An insight into the effects of these parameters of the flow field can be obtained by the study of the temperature and mass concentration distributions.

The dimensionless temperature $\theta(\eta)$ and dimensionless mass concentration $\phi(\eta)$ have been plotted against the dimension $\eta$ for several sets of the values of the parameters $K$, $\alpha$, $c$, $\gamma$, $\Pr$, $S_c$ and $\theta_e$.

In Fig 1 we are observing the effect of concentration profile with the variation of Schmidt number $S_c$. The values of $S_c = 1.00,\ 2.25,\ 2.75,\ 4.50,\ 5.50$ with the values of other parameters $\Pr = 3.70$, $K = 2.00$, $\alpha = 0.50$, $\gamma = 2.00$, $c = 0.25$ and the thermal conductivity parameter $\theta_e$=-10. A rise in $S_c$ strongly suppreses concentration levels in the boundary layer regime. All profiles decay monotonically from the surface (wall) to the free stream. $S_c$ embodies the ratio of momentum diffusivity ($\upsilon$) to molecular diffusivity($D$). It is observed that the fluids concentration decreases as the mass transfer parameters $S_c$ increases.

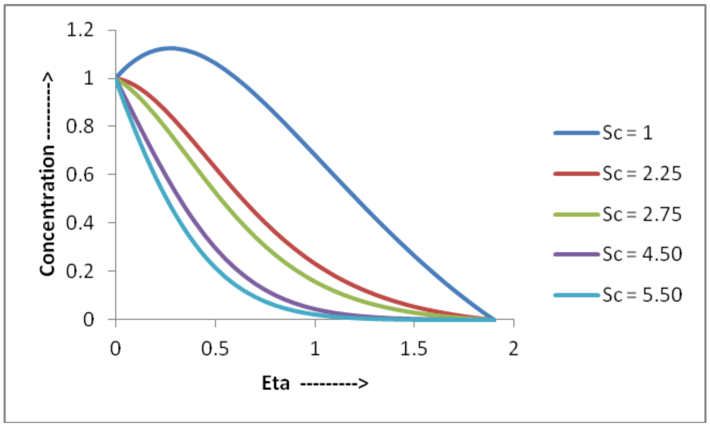


Fig 1: Effect of Schmidt number $S_c$ on concentration.

In Fig 2 we study the effect of $\theta_e$ the variable viscosity prameter on concentration profile. The values of $\theta_e$ = -15, -12, -10 has been considered and the other parameters are taken as $\Pr = 3.70$, $K = 2.00$, $\alpha = 0.50$, $\gamma = 2.00$, $c = 0.25$ and $S_c = 2.50$. It is observed that the concentration profile decreases with the increase of the variable viscosity parameter $\theta_e$.

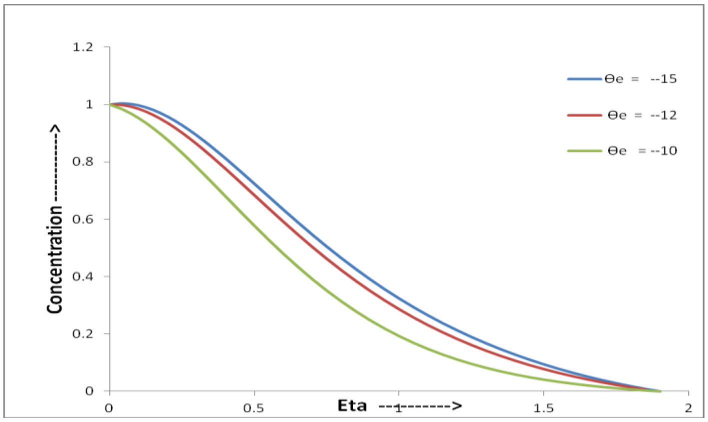


Fig 2: Effect of variable viscosity $\theta_e$ on concentration.

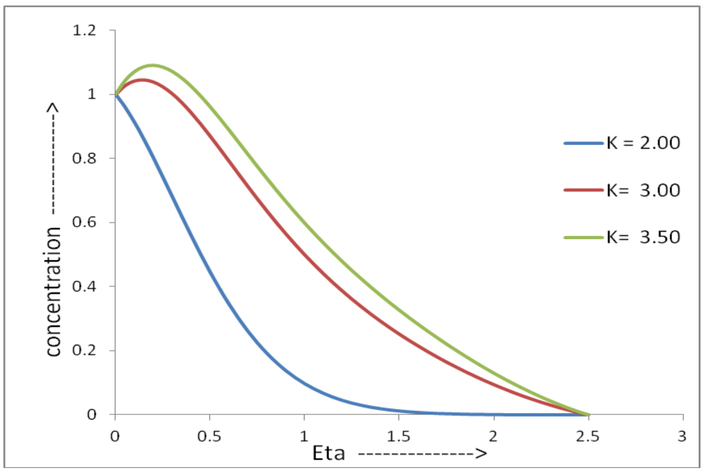


Fig 3: Effect of viscoelastic parameter $K$ on concentration.

Fig 3 illustrates the effects of the viscoelastic parameter $K$ on the concentration profile. Substituting various values of $K$ =2.00, 3.00, 3.50 at $\Pr = 3.70$, $S_c = 2.50$, $\alpha = 0.50$, $\gamma = 2.00$, $c = 0.50$ and $\theta_e$ =-10, it is observed that the concentration profile increases with the increase of viscoelastic parameter $K$ .

In Fig 4, it has been investigated that the concentration profile increases with the change of Prandtl number Pr. The study revels that concentration increases with the increase of Pr.

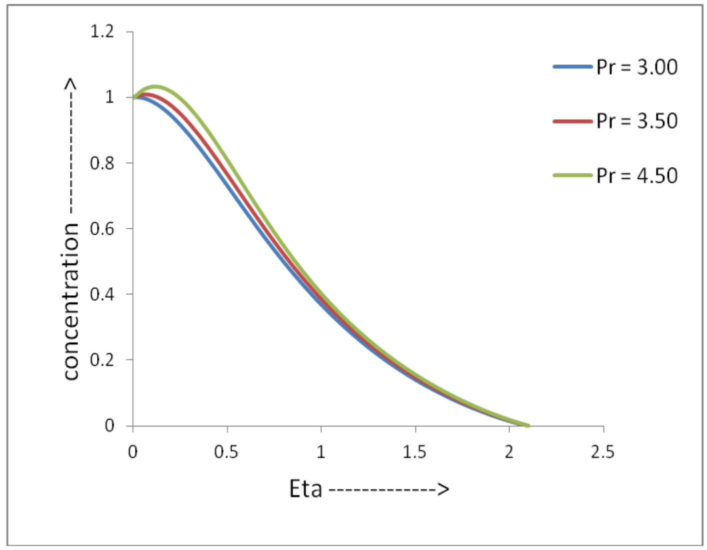


Fig 4: Variation of concentration profile with Pr.

In Fig 5, the temperature profile for various values of Pr has been studied. It is observed that the temperature profile decreases with the increase of the Prandtl number.

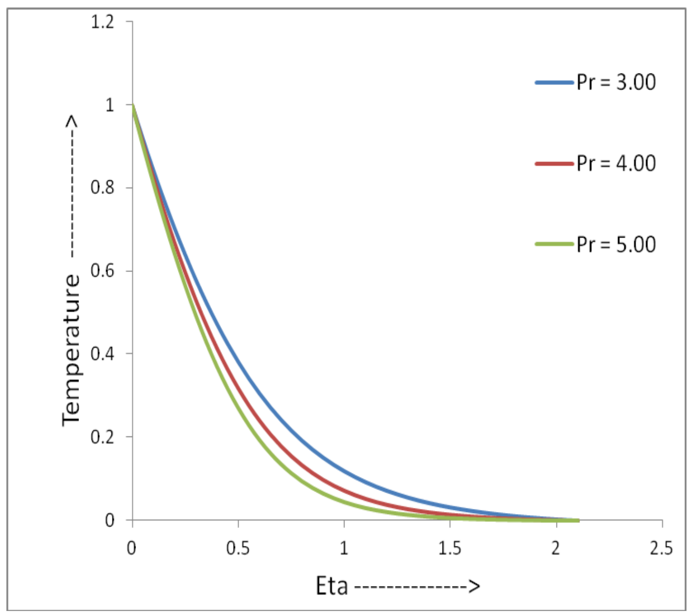


Fig 5: Variation of temperature profile with Pr.

Figures 6 and 7 exhibits the effects of heat parameter $\alpha$ on temperature distribution. It is evident from the Fig 6, the temperature distribution $\theta(\eta)$ increases with an increase in the heat parameter +ve $\alpha$ and the inverse is true seing in Fig 7.

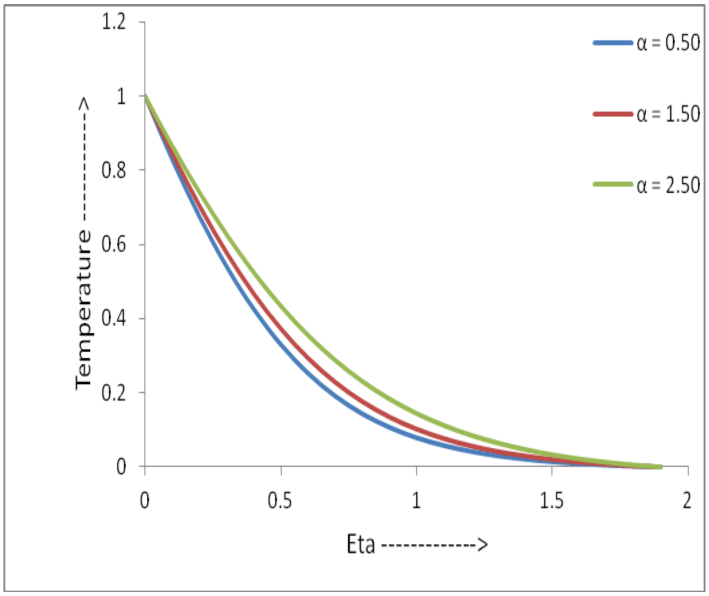


Fig 6: Temperature distribution for varies values of the heat parameter $\alpha$ (positive).

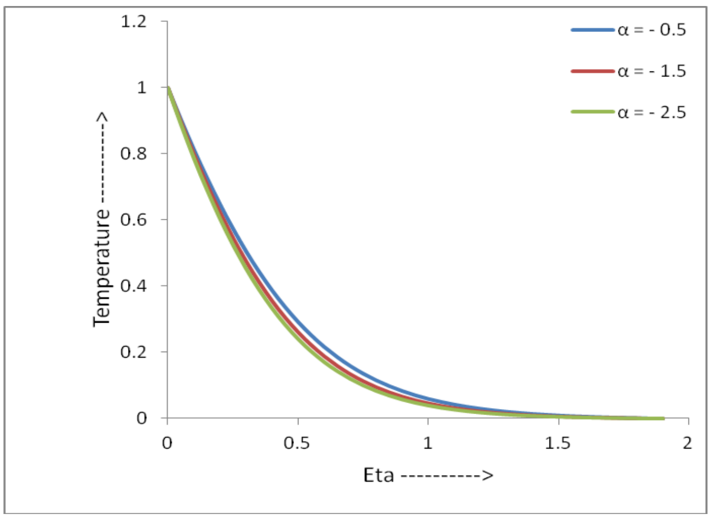


Fig 7: Temperature distribution for varies values of the heat parameter $\alpha$ (negative).

In the Fig 8, the effect of dimensionless stretching ratio $c$ on concentration profile has been studied. It is observed that the concentration profile decreases with the increase of starching parameter.

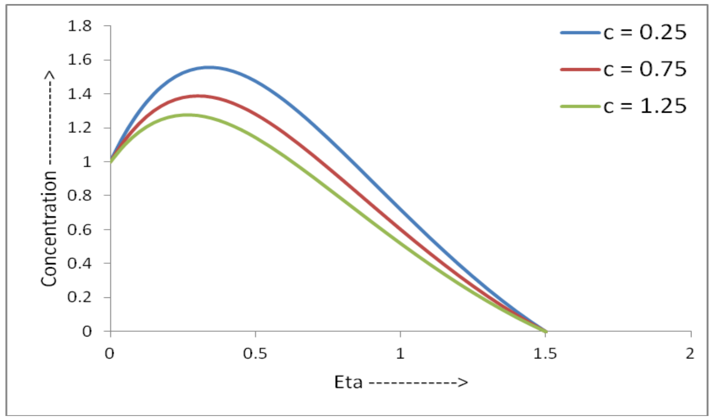


Fig 8: Concertration distribution for varies values of the dimensionless stretching ratio $c$.

In the Fig 9 we observe the effect of chemical reaction parameter $\gamma$ on concentration profile $\phi$. Substituting various values for $\Pr = 3.70$, $K = 2.00$, $\alpha = 0.50$, $S_c = 2.50$, $c = 0.25$ and the thermal conductivity parameter $\theta_e$=-10, it is observed that the concentration profile increases with the increase of reaction parameter $\gamma$ of the fluid flow.

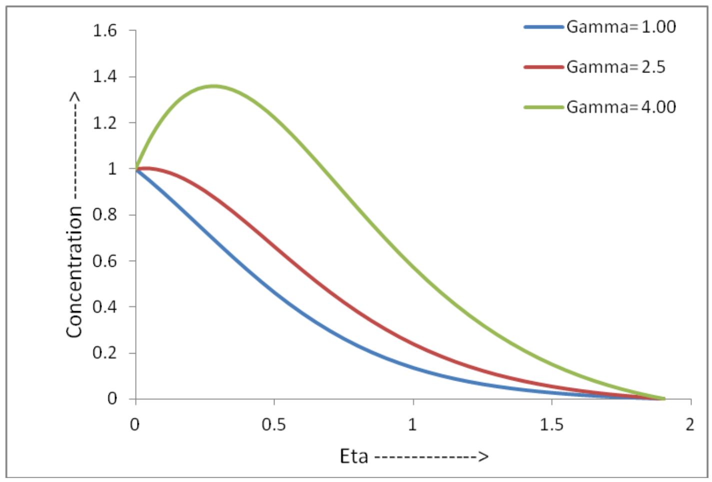


Fig 9: Concentration distribution for varies values of the Chemical reaction parameter $\gamma$.

Missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for various values of $\theta_e$, $K$, $\alpha$, $\gamma$ and $c$ have been derived. In Table I, missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ were found for $\theta_e$ = -15, -14, -13, -12, -11, -10, -9, -8, -7, -6, -5, -4. It is observed that the missing values of $f''(0)$ decreases while that of $\theta'(0)$ and $\phi'(0)$ increases. In Table II we observed the missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for $K$ = 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6. The study revels that the missing values of $f''(0)$ increases while that of $\theta'(0)$ and $\phi'(0)$ decreases.

In Table III it is observed that for increasing values of $c$ = 0, 0.25, 0.50, 0.75, 1.00, 1.25 the missing values of $f''(0)$ increases while that of $\theta'(0)$ and $\phi'(0)$ decrease.

Table I: Missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for various values of $\theta_e$

| $\theta_e$ | $f''(0)$ | $\theta'(0)$ | $\phi'(0)$ |
|---|---|---|---|
| -15 | -0.908027 | -1.694586 | -0.157126 |
| -14 | -0.908397 | -1.694487 | -0.156923 |
| -13 | -0.908821 | -1.694373 | -0.156691 |
| -12 | -0.909312 | -1.694241 | -0.156423 |
| -11 | -0.909887 | -1.694087 | -0.156107 |
| -10 | -0.910571 | -1.693904 | -0.155731 |
| -9 | -0.911396 | -1.693683 | -0.155281 |
| -8 | -0.912412 | -1.69341 | -0.154724 |
| -7 | -0.913694 | -1.693067 | -0.154021 |
| -6 | -0.915363 | -1.69262 | -0.153108 |
| -5 | -0.917623 | -1.692016 | -0.151872 |
| -4 | -0.920858 | -1.691152 | -0.150104 |

Table II: Missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for various values of $K$.

| $K$ | $f''(0)$ | $\theta'(0)$ | $\phi'(0)$ |
|---|---|---|---|
| 2.00 | -0.633191 | -1.82256 | 0.150568 |
| 2.50 | -0.628199 | -1.823425 | 0.144856 |
| 3.00 | -0.625143 | -1.823882 | 0.14207 |
| 3.50 | -0.623027 | -1.824177 | 0.140342 |
| 4.00 | -0.621465 | -1.824385 | 0.139152 |
| 4.50 | -0.62026 | -1.824539 | 0.138266 |
| 5.00 | -0.619301 | -1.824659 | 0.137588 |
| 5.50 | -0.618520 | -1.824755 | 0.137053 |
| 6.00 | -0.617871 | -1.824832 | 0.136615 |

Table III: Missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for various values of $c$

| $c$ | $f''(0)$ | $\theta'(0)$ | $\phi'(0)$ |
|---|---|---|---|
| 0 | -0.733158 | -1.689918 | 1.487715 |
| 0.25 | -0.722720 | -1.743844 | 1.353018 |
| 0.50 | -0.708433 | -1.795084 | 1.237238 |
| 0.75 | -0.694258 | -1.843970 | 1.134856 |
| 1.00 | -0.680144 | -1.891197 | 1.040672 |
| 1.25 | -0.669834 | -1.933683 | 0.973830 |

## 4 Conclusion

A numerical study of the effect of variable viscosity on boundary layer flow of second order fluids over a stretching surface with mass and heat transfer has been offered.

The subsequent outcome may be drawn as:

1. The temperature profile within the boundary flim rises for the decreasing values of Prandtl number $\Pr$ .
2. The concentration profile within the boundary flim raises considerable for the increasing values of $\Pr$ , $K$ , $\gamma$ and decreases for $\theta_e$ .
3. The temperature profile within the boundary flim raises considerable for the increasing values of heat (Source/sink) parameter $\alpha$ .

## 5 Acknowledgments

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